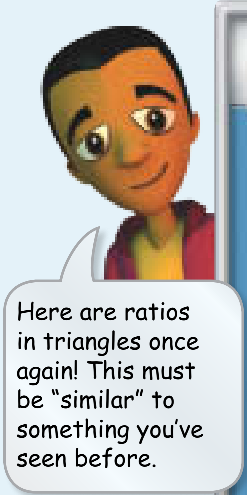
8-3 Trigonometry

Common Core State Standards
G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Also, G-SRT.C.7, G-MG.A.1**
MP 1, MP 3, MP 4, MP 6

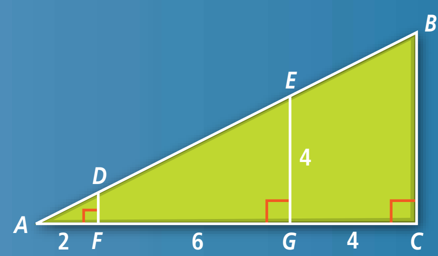
Objective To use the sine, cosine, and tangent ratios to determine side lengths and angle measures in right triangles



MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

What is the ratio of the length of the shorter leg to the length of the hypotenuse for each of $\triangle ADF$, $\triangle AEG$, and $\triangle ABC$? Make a conjecture based on your results.



Essential Understanding If you know certain combinations of side lengths and angle measures of a right triangle, you can use ratios to find other side lengths and angle measures.

Lesson Vocabulary
- trigonometric ratios
- sine
- cosine
- tangent

Any two right triangles that have a pair of congruent acute angles are similar by the AA Similarity Postulate. Similar right triangles have equivalent ratios for their corresponding sides called **trigonometric ratios.**

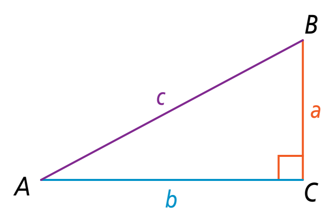
take note

Key Concept Trigonometric Ratios

sine of $\angle A = \dfrac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \dfrac{a}{c}$

cosine of $\angle A = \dfrac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \dfrac{b}{c}$

tangent of $\angle A = \dfrac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \dfrac{a}{b}$



You can abbreviate the ratios as

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}},\ \cos A = \frac{\text{adjacent}}{\text{hypotenuse}},\ \text{and } \tan A = \frac{\text{opposite}}{\text{adjacent}}.$$

Think

How do the sides relate to $\angle T$?
$\overline{GR}$ is across from, or *opposite*, $\angle T$. $\overline{TR}$ is next to, or *adjacent* to, $\angle T$. $\overline{TG}$ is the *hypotenuse* because it is opposite the 90° angle.

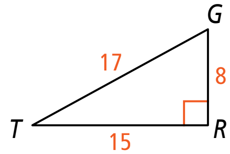
Problem 1 Writing Trigonometric Ratios

What are the sine, cosine, and tangent ratios for $\angle T$?

$$\sin T = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$$

$$\cos T = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$$

$$\tan T = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$$



Got It? **1.** Use the triangle in Problem 1. What are the sine, cosine, and tangent ratios for $\angle G$?

In Chapter 7, you used similar triangles to measure distances indirectly. You can also use trigonometry for indirect measurement.

Problem 2 Using a Trigonometric Ratio to Find Distance

Landmarks **In 1990, the Leaning Tower of Pisa was closed to the public due to safety concerns. The tower reopened in 2001 after a 10-year project to reduce its tilt from vertical. Engineers' efforts were successful and resulted in a tilt of 5°, reduced from 5.5°. Suppose someone drops an object from the tower at a height of 150 ft. How far from the base of the tower will the object land? Round to the nearest foot.**

Plan

What is the first step?
Look at the triangle and determine how the sides of the triangle relate to the given angle.

The given side is adjacent to the given angle. The side you want to find is opposite the given angle.

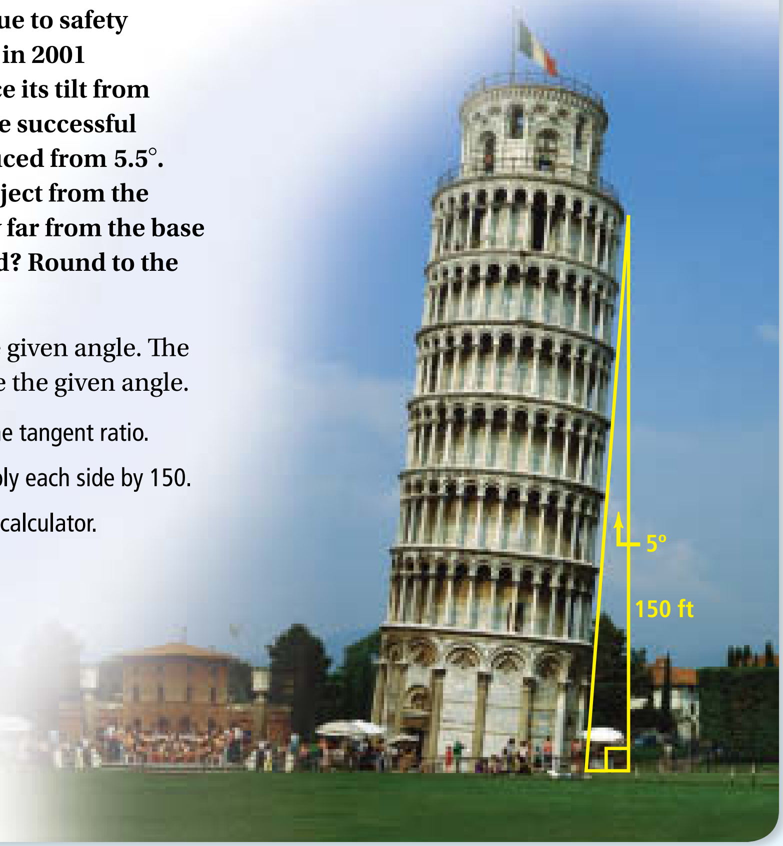
$\tan 5^\circ = \frac{x}{150}$ Use the tangent ratio.

$x = 150(\tan 5^\circ)$ Multiply each side by 150.

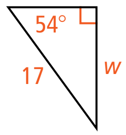
150 tan 5 enter Use a calculator.

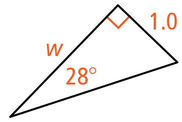
$x \approx 13.12329953$

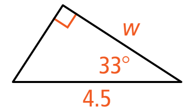
The object will land about 13 ft from the base of the tower.



Got It? **2.** For parts (a)–(c), find the value of w to the nearest tenth.

a.


b.


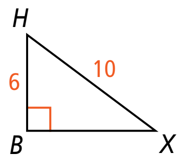
c.


d. A section of Filbert Street in San Francisco rises at an angle of about 17°. If you walk 150 ft up this section, what is your vertical rise? Round to the nearest foot.

If you know the sine, cosine, or tangent ratio for an angle, you can use an inverse $(\sin^{-1}, \cos^{-1}, \text{or } \tan^{-1})$ to find the measure of the angle.

Problem 3 Using Inverses

What is $m\angle X$ to the nearest degree?

A


You know the lengths of the hypotenuse and the side opposite $\angle X$.

Use the sine ratio.

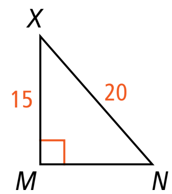
$\sin X = \frac{6}{10}$ Write the ratio.

$m\angle X = \sin^{-1}\left(\frac{6}{10}\right)$ Use the inverse.

sin-1 6 ÷ 10 enter Use a calculator.

$m\angle X \approx 36.86989765$

≈ 37

B


You know the lengths of the hypotenuse and the side adjacent to $\angle X$.

Use the cosine ratio.

$\cos X = \frac{15}{20}$

$m\angle X = \cos^{-1}\left(\frac{15}{20}\right)$

cos-1 15 ÷ 20 enter

$m\angle X \approx 41.40962211$

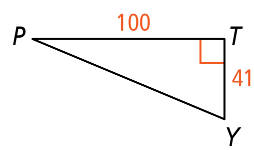
≈ 41

Think

When should you use an inverse?
Use an inverse when you know two side lengths of a right triangle and you want to find the measure of one of the acute angles.

Got It? **3. a.** Use the figure at the right. What is $m\angle Y$ to the nearest degree?

b. Reasoning Suppose you know the lengths of all three sides of a right triangle. Does it matter which trigonometric ratio you use to find the measure of any of the three angles? Explain.



Lesson Check

Do you know HOW?

Write each ratio.

1. sin A

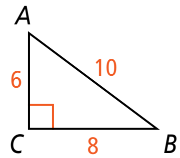
2. cos A

3. tan A

4. sin B

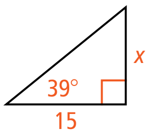
5. cos B

6. tan B



What is the value of x? Round to the nearest tenth.

7.



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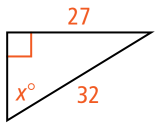
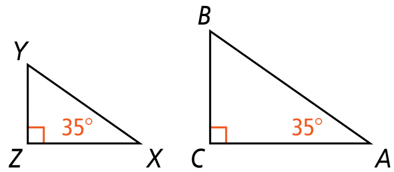


Do you UNDERSTAND? MATHEMATICAL PRACTICES

9. Vocabulary Some people use SOH-CAH-TOA to remember the trigonometric ratios for sine, cosine, and tangent. Why do you think that word might help? (*Hint*: Think of the first letters of the ratios.)

10. Error Analysis A student states that sin $A >$ sin X because the lengths of the sides of $\triangle ABC$ are greater than the lengths of the sides of $\triangle XYZ$. What is the student's error? Explain.



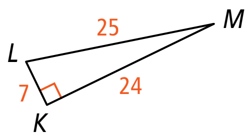
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Write the ratios for sin M, cos M, and tan M.

See Problem 1.

11.



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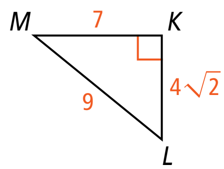

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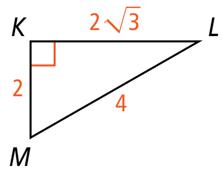
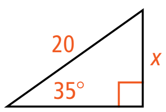


Find the value of x. Round to the nearest tenth.

See Problem 2.

14.



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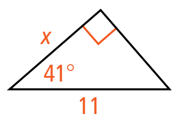

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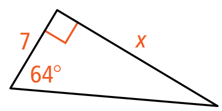

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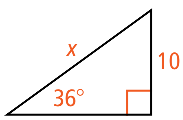

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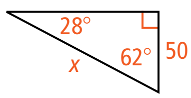

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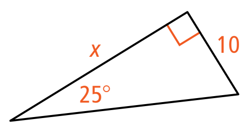


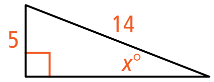
20. Recreation A skateboarding ramp is 12 in. high and rises at an angle of 17°. How long is the base of the ramp? Round to the nearest inch.

21. Public Transportation An escalator in the subway station has a vertical rise of 195 ft 9.5 in., and rises at an angle of 10.4°. How long is the escalator? Round to the nearest foot.

Find the value of x. Round to the nearest degree.

See Problem 3.

22.



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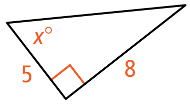

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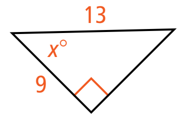

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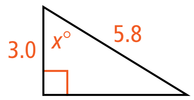

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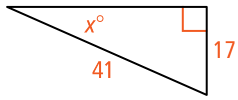

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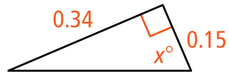
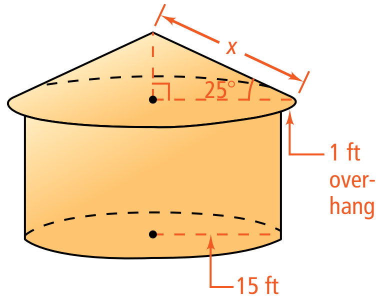


B Apply

28. The lengths of the diagonals of a rhombus are 2 in. and 5 in. Find the measures of the angles of the rhombus to the nearest degree.

29. Think About a Plan Carlos plans to build a grain bin with a radius of 15 ft. The recommended slant of the roof is 25°. He wants the roof to overhang the edge of the bin by 1 ft. What should the length x be? Give your answer in feet and inches.

- What is the position of the side of length x in relation to the given angle?
- What information do you need to find a side length of a right triangle?
- Which trigonometric ratio could you use?



An *identity* is an equation that is true for all the allowed values of the variable. Use what you know about trigonometric ratios to show that each equation is an identity.

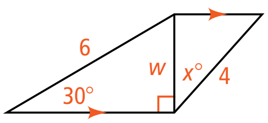
30. $\tan X = \dfrac{\sin X}{\cos X}$

31. $\sin X = \cos X \cdot \tan X$

32. $\cos X = \dfrac{\sin X}{\tan X}$

Find the values of w and then x. Round lengths to the nearest tenth and angle measures to the nearest degree.

33.



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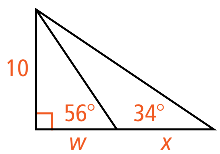

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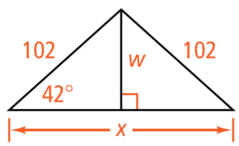
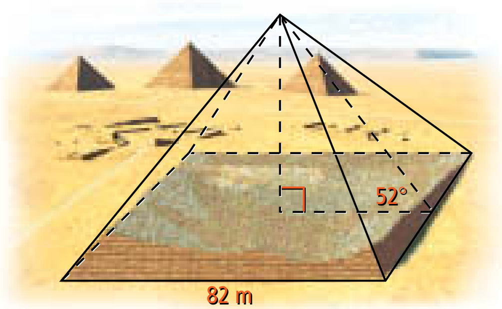


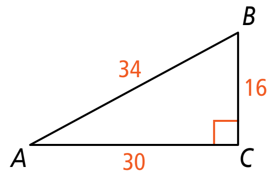
STEM **36. Pyramids** All but two of the pyramids built by the ancient Egyptians have faces inclined at 52° angles. Suppose an archaeologist discovers the ruins of a pyramid. Most of the pyramid has eroded, but the archaeologist is able to determine that the length of a side of the square base is 82 m. How tall was the pyramid, assuming its faces were inclined at 52°? Round your answer to the nearest meter.



37. a. In $\triangle ABC$ at the right, how does $\sin A$ compare to $\cos B$? Is this true for the acute angles of other right triangles?

b. **Reading Math** The word cosine is derived from the words *complement's sine*. Which angle in $\triangle ABC$ is the complement of $\angle A$? Of $\angle B$?

c. Explain why the derivation of the word cosine makes sense.



38. For right $\triangle ABC$ with right $\angle C$, prove each of the following.

Proof

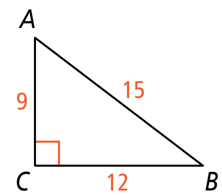
a. $\sin A < 1$

b. $\cos A < 1$

39. a. **Writing** Explain why $\tan 60° = \sqrt{3}$. Include a diagram with your explanation.

b. **Make a Conjecture** How are the sine and cosine of a $60°$ angle related? Explain.

The sine, cosine, and tangent ratios each have a reciprocal ratio. The reciprocal ratios are cosecant (csc), secant (sec), and cotangent (cot). Use $\triangle ABC$ and the definitions below to write each ratio.



$\csc X = \dfrac{1}{\sin X}$ $\qquad$ $\sec X = \dfrac{1}{\cos X}$ $\qquad$ $\cot X = \dfrac{1}{\tan X}$

40. $\csc A$

41. $\sec A$

42. $\cot A$

43. $\csc B$

44. $\sec B$

45. $\cot B$

46. **Graphing Calculator** Use the table feature of your graphing calculator to study sin X as X gets close to (but not equal to) 90. In the y= screen, enter Y1 = sin X.

a. Use the tblset feature so that X starts at 80 and changes by 1. Access the table. From the table, what is sin X for X = 89?

b. Perform a "numerical zoom-in." Use the tblset feature, so that X starts with 89 and changes by 0.1. What is sin X for X = 89.9?

c. Continue to zoom-in numerically on values close to 90. What is the greatest value you can get for sin X on your calculator? How close is X to 90? Does your result contradict what you are asked to prove in Exercise 38a?

d. Use right triangles to explain the behavior of sin X found above.

47. a. **Reasoning** Does $\tan A + \tan B = \tan(A + B)$ when $A + B < 90$? Explain.

b. Does $\tan A - \tan B = \tan(A - B)$ when $A - B > 0$? Use part (a) and indirect reasoning to explain.

Challenge

Verify that each equation is an identity by showing that each expression on the left simplifies to 1.

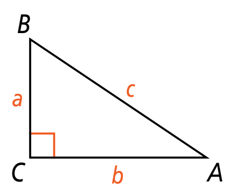
48. $(\sin A)^2 + (\cos A)^2 = 1$

49. $(\sin B)^2 + (\cos B)^2 = 1$

50. $\dfrac{1}{(\cos A)^2} - (\tan A)^2 = 1$

51. $\dfrac{1}{(\sin A)^2} - \dfrac{1}{(\tan A)^2} = 1$

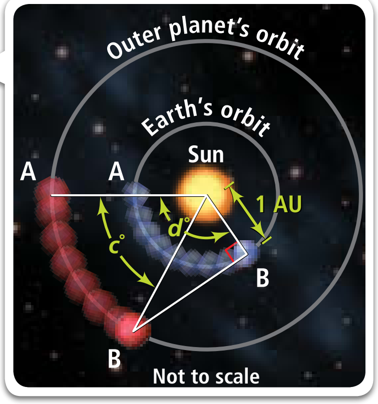
52. Show that $(\tan A)^2 - (\sin A)^2 = (\tan A)^2 \cdot (\sin A)^2$ is an identity.



STEM **53. Astronomy** The Polish astronomer Nicolaus Copernicus devised a method for determining the sizes of the orbits of planets farther from the sun than Earth. His method involved noting the number of days between the times that a planet was in the positions labeled A and B in the diagram. Using this time and the number of days in each planet's year, he calculated c and d.

a. For Mars, $c = 55.2$ and $d = 103.8$. How far is Mars from the sun in astronomical units (AU)? One astronomical unit is defined as the average distance from Earth to the center of the sun, about 93 million miles.

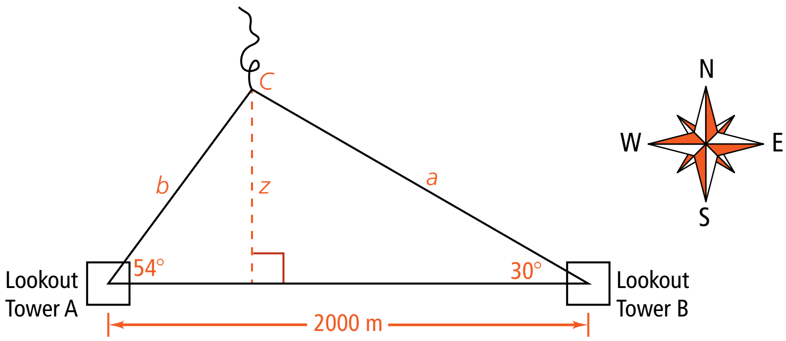
b. For Jupiter, $c = 21.9$ and $d = 100.8$. How far is Jupiter from the sun in astronomical units?



Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1

Look back at the information on page 489 about the fire in a state forest. The diagram is shown again below.



Select all of the following that are true. Explain your reasoning.

A. $\sin 54° = \frac{z}{b}$

B. $\cos 30° = \frac{z}{2000}$

C. $\tan 30° = \frac{z}{2000}$

D. $\sin 30° = \frac{z}{a}$

E. $\tan 54° = \frac{z}{2000}$

F. $\cos 54° = \frac{z}{b}$